Mikko.Pietilä *Assignment.5* *Calculations*

$F := (2 \cdot 1900\ kg \cdot g) + 18\ kg \cdot g = 37.442\ kN$

$L.1 := 2.400\ m$ $\quad h := 120\ mm$

$L.2 := 1.700\ m$ $\quad t := 15\ mm$ $\quad b := 120\ mm$

$L.3 := L.1 \cdot \tan(50\ deg) = 2.86\ m$ $\quad \sigma_{yield} := 355\ MPa$

$L.3 := 2.860\ m$

$E := 190\ GPa$ $\quad max.deflection\ \ L.3 \cdot 0.2\% = 5.72\ mm$

$Safety := 2$

$$\frac{F \cdot L.2^3}{6 \cdot E \cdot I}\left(\frac{3 \cdot L.3}{L.2} - 1\right) - \frac{By \cdot L.3^3}{3 \cdot E \cdot I} = 0 \xrightarrow{solve,\,By} 15.911634183281139996 \cdot kN$$

$$I_z := \frac{b \cdot h^3}{12} - \frac{(b-t) \cdot (h - 2 \cdot t)^3}{12} = (1.09 \cdot 10^7)\ mm^4$$

$Beam.weight := 38.86\ \frac{kg}{m}$

$Beam.weight \cdot g = 0.381\ \frac{1}{m} \cdot kN$

$q := 0.381\ \frac{kN}{m}$

$$By := \frac{F \cdot L.2^3}{6 \cdot E \cdot I} \cdot \left(\frac{3 \cdot L.3}{L.2} - 1\right) + \frac{q \cdot L.3^4}{8 \cdot E \cdot I} - \frac{By \cdot L.3^3}{3 \cdot E \cdot I} = 0 \xrightarrow{solve,\,By} 0.4086225 \cdot kN + 15.911634183281139996 \cdot kN = 16.32\ kN$$

$B := \frac{By}{\cos(50\ deg)} = 25.39\ kN$

$Bx := B \cdot \sin(50\ deg) = 19.45\ kN$

$M.A := -F \cdot L.2 - q \cdot L.3 \cdot \frac{L.3}{2} + By \cdot L.3 = -18.533\ kN \cdot m$

$Ay := F + q \cdot L.3 - By = 22.211\ kN$

$Ax := Bx = 19.45\ kN$

$$\sigma_x := \frac{Ax}{h \cdot b - (h - 2 \cdot t) \cdot (b - 2 \cdot t)} = 3.087\ MPa$$

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$$\sigma_{bending} := \frac{-M.A \cdot \frac{h}{2}}{I_z} = 102.007\ MPa$$

$$\sigma_{yield} - Safety \cdot \sigma_{bending} = 150.987\ MPa$$

$$v(x) := \begin{cases} \text{if } 0\ m \le x \le L.2 \\ \quad \frac{F}{6 \cdot E \cdot I_z} \cdot \left(3 \cdot L.2 \cdot x^2 - x^3\right) + \frac{q}{24 \cdot E \cdot I_z} \cdot \left(6 \cdot L.3^2 \cdot x^2 - 4 \cdot L.3 \cdot x^3 + x^4\right) - \frac{By}{6 \cdot E \cdot I_z} \cdot \left(3 \cdot L.3 \cdot x^2 - x^3\right) \\ \text{if } L.2 < x \le L.3 \\ \quad \frac{F}{6 \cdot E \cdot I_z} \cdot \left(3 \cdot L.2 \cdot x^2 - x^3 + (x - L.2)^3\right) + \frac{q}{24 \cdot E \cdot I_z} \cdot \left(6 \cdot L.3^2 \cdot x^2 - 4 \cdot L.3 \cdot x^3 + x^4\right) - \frac{By}{6 \cdot E \cdot I_z} \cdot \left(3 \cdot L.3 \cdot x^2 - x^3\right) \end{cases}$$

$$f(x) := \frac{F}{6 \cdot E \cdot I_z} \cdot \left(3 \cdot L.2 \cdot x^2 - x^3 + (x - L.2)^3\right) + \frac{q}{24 \cdot E \cdot I_z} \cdot \left(6 \cdot L.3^2 \cdot x^2 - 4 \cdot L.3 \cdot x^3 + x^4\right) - \frac{By}{6 \cdot E \cdot I_z} \cdot \left(3 \cdot L.3 \cdot x^2 - x^3\right)$$

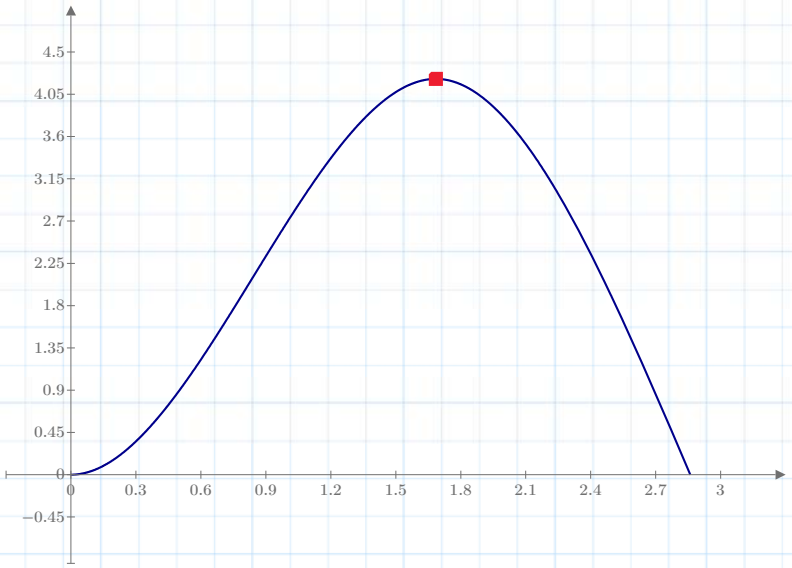
$$g(x) := \frac{d}{dx} f(x)$$

$$g(x) = 0 \xrightarrow{assume,\ solve,\ x} ? = \begin{bmatrix} 4.024 \cdot 10^3 + 1.421i \cdot 10^{-11} \\ 1.685 \cdot 10^3 - 1.421i \cdot 10^{-11} \\ -1.256 \cdot 10^5 + 1.221i \cdot 10^{-12} \end{bmatrix} mm$$

$$x := 0, 0.001\ m .. L.3$$

$$x_{maximum} := 1.685 \cdot 10^3\ mm$$

$$v\left(x_{maximum}\right) = 4.214\ mm \qquad max.deflection$$



$v(x)\ (mm)$

$v\left(x_{maximum}\right)\ (mm)$

$x\ (m)$

$x_{maximum}\ (m)$

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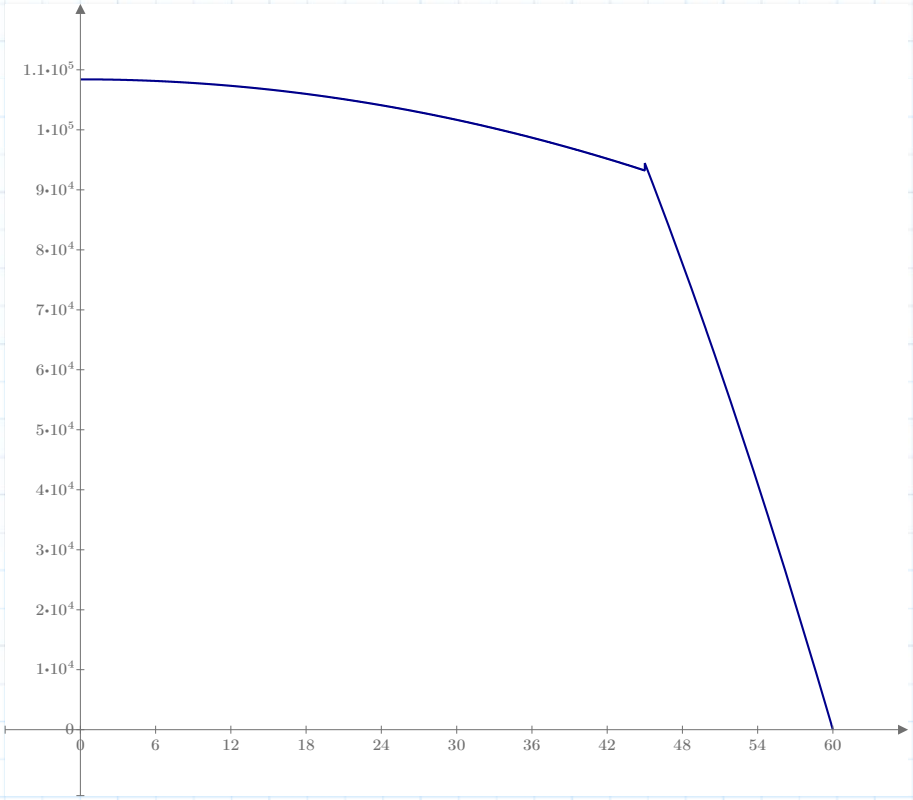
Buckling

$$P_{cr} := \frac{2.046 \cdot \pi^2 \cdot E \cdot I_z}{L.3^2} = \left(5.113 \cdot 10^3\right)\ kN \qquad P_{cr} - Bx = \left(5.094 \cdot 10^3\right)\ kN$$

Shear.stresses

$$y_1 := 0, 0.01\ mm .. \frac{h}{2}$$

$$Q\left(y_1\right) := \left\| \begin{array}{l} \text{if } 0 \le y_1 \le \left(\frac{h}{2} - t\right) \\ \quad \left\| b \cdot t \cdot \left(\frac{h}{2} - \frac{t}{2}\right) + \frac{t \cdot \left(h^2 - 4 \cdot h \cdot t + t^2 - 4 \cdot {y_1}^2\right)}{8} \right. \\ \text{if } \left(\frac{h}{2} - t\right) < y_1 \le \frac{h}{2} \\ \quad \left\| b \cdot \left(\frac{h}{2} - y_1\right) \cdot \left(y_1 + \frac{\left(\frac{h}{2} - y_1\right)}{2}\right) \right. \end{array} \right.$$



$Q\left(y_1\right)\ \left(mm^3\right)$

$\left(y_1\right)\ \left(mm\right)$

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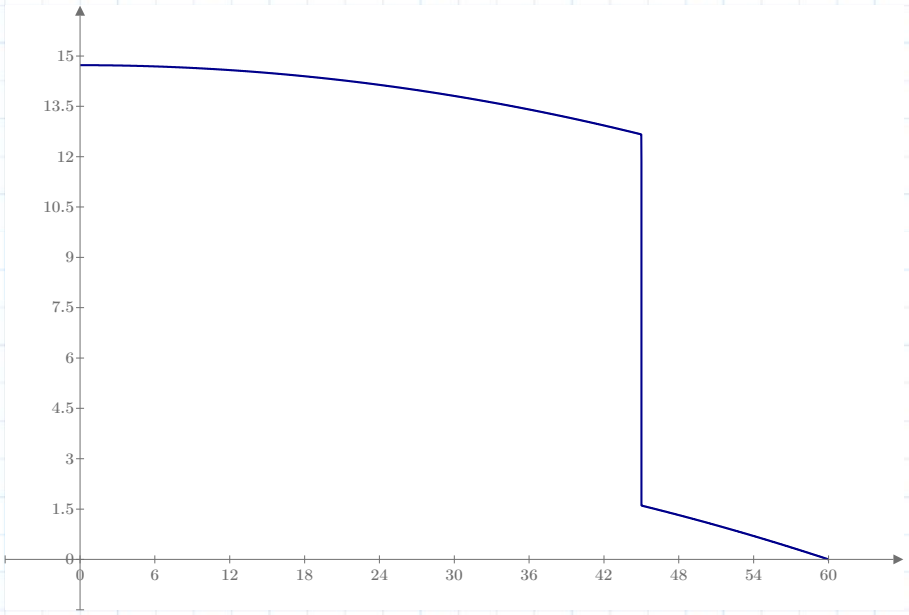
Welding $t_w := 8\ mm$

$$\tau_{xy}(y_1) := \begin{cases} \text{if } 0 \le y_1 < \left(\frac{h}{2} - t\right) \\ \quad \frac{Ay \cdot Q(y_1)}{t \cdot I_z} \\ \text{if } \left(\frac{h}{2} - t\right) = y_1 \\ \quad \frac{Ay \cdot Q(y_1)}{2 \cdot t_w \cdot I_z} \\ \text{if } \left(\frac{h}{2} - t\right) < y_1 \le \frac{h}{2} \\ \quad \frac{Ay \cdot Q(y_1)}{b \cdot I_z} \end{cases}$$



$\tau_{xy}(y_1)$ (MPa)

y_1 (mm)

$$\tau_w := \tau_{xy}\left(\frac{h}{2} - t\right) = 11.873\ MPa$$

$$\tau_{middle} := \frac{Ay \cdot \left(t \cdot b \cdot \left(\frac{h}{2} - \frac{t}{2}\right) + \left(\frac{h}{2} - t\right) \cdot t \cdot \frac{\left(\frac{h}{2} - t\right)}{2}\right)}{t \cdot I_z} = 14.899\ MPa$$

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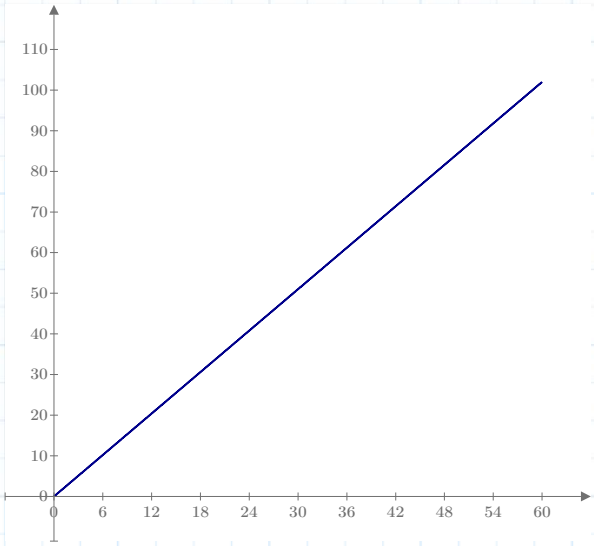
$$\sigma_{bending}(y_1) := \frac{-M.A \cdot y_1}{I_z}$$

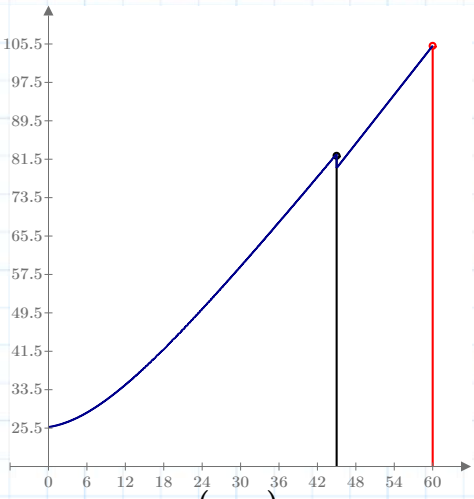


$\sigma_{bending}(y_1)$ (MPa)

y_1 (mm)

Vonmises *Compression*

$$\sigma_{Vmcompression}(y_1) := \sqrt{(\sigma_{bending}(y_1) + \sigma_x)^2 + 3 \cdot \tau_{xy}(y_1)^2}$$



$\sigma_{Vmcompression}(y_1)$ (MPa)

$\sigma_{Vmcompression}\left(\frac{h}{2} - t\right)$ (MPa)

$\sigma_{Vmcompression}\left(\frac{h}{2}\right)$ (MPa)

y_1 (mm)

$\frac{h}{2} - t$ (mm)

$\frac{h}{2}$ (mm)

$$\sigma_{maximum} := \sigma_{Vmcompression}\left(\frac{h}{2}\right) = 105.094 \ MPa$$

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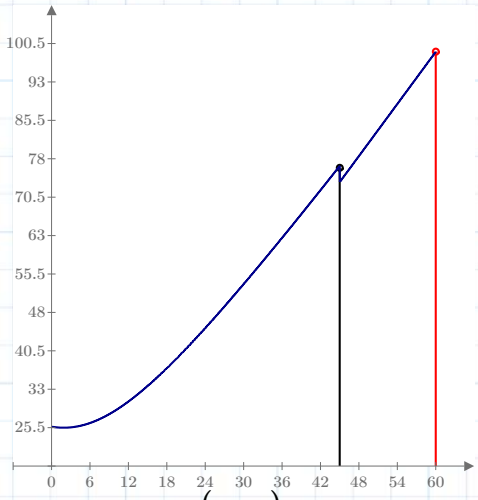
Vonmises *Tension*

$$\sigma_{VmTension}(y_1) := \sqrt{(\sigma_{bending}(y_1) - \sigma_x)^2 + 3 \cdot \tau_{xy}(y_1)^2}$$



y_1 (mm)

$\sigma_{VmTension}(y_1)$ (MPa)

$\frac{h}{2} - t$ (mm)

$\sigma_{VmTension}\left(\frac{h}{2} - t\right)$ (MPa)

$\frac{h}{2}$ (mm)

$\sigma_{VmTension}\left(\frac{h}{2}\right)$ (MPa)

$\sigma_{VmTension}\left(\frac{h}{2} - t\right) = 76.243$ MPa

$\sigma_{VmTension}\left(\frac{h}{2}\right) = 98.919$ MPa

$\sigma_{yield} - \sigma_{maximum} \cdot 2 = 144.812$ MPa

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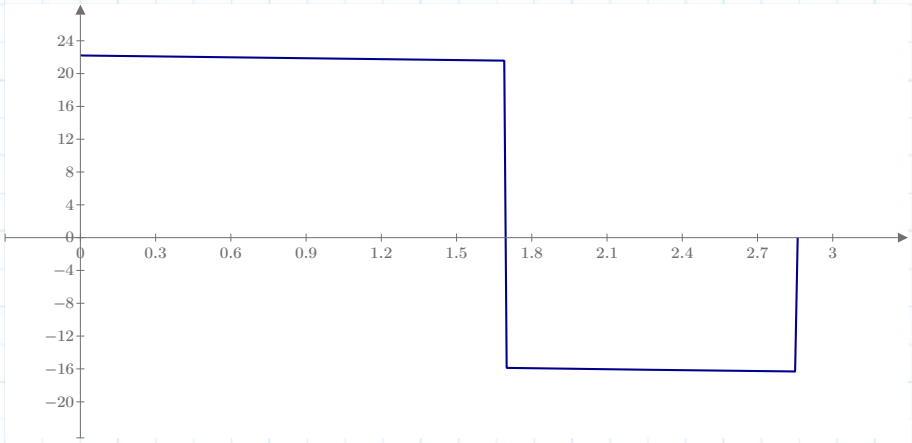
ShearForce.and.Moment.Diagrams

$z := 0, 0.01\ \mathbf{m} .. L.3$

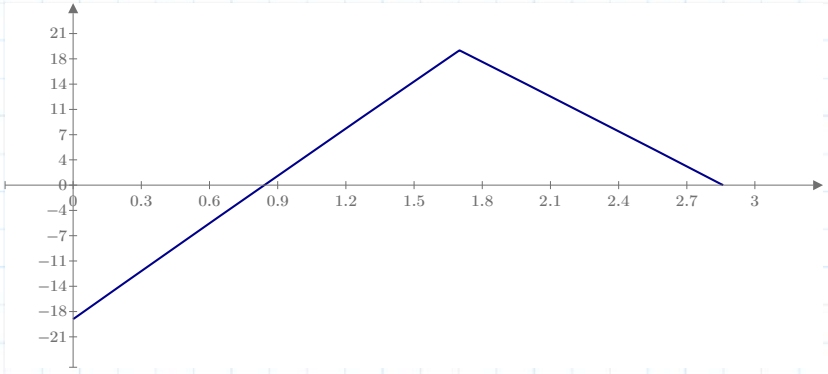
$$V(z) := \left\| \begin{array}{l} \text{if } 0\ \mathbf{m} \le z < L.2 \\ \quad \| Ay - q \cdot z \\ \text{if } L.2 \le z < L.3 \\ \quad \| Ay - q \cdot z - F \\ \text{if } L.3 = z \\ \quad \| Ay - q \cdot z - F + By \end{array} \right.$$



$V(z)\ (\mathbf{kN})$

$(z)\ (\mathbf{m})$

$$M(z) := \left\| \begin{array}{l} \text{if } 0\ \mathbf{m} \le z < L.2 \\ \quad \left\| M.A + Ay \cdot z - \frac{q \cdot z^2}{2} \right. \\ \text{if } L.2 \le z \le L.3 \\ \quad \left\| M.A + Ay \cdot z - \frac{q \cdot z^2}{2} - F \cdot (z - L.2) \right. \end{array} \right.$$



$M(z)\ (\mathbf{kN} \cdot \mathbf{m})$

$(z)\ (\mathbf{m})$

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